The analysis of optical waves can be broken up into different regimes.

- **Quantum Optics**
- **Electromagnetic Optics**
- **Wave Optics**
- **Ray Optics**

Historically, optical theory developed as:

Ray Optics → Wave Optics → Electromagnetic Optics → Quantum Optics

**Quantum Optics**: The interaction of photons using Quantum Mechanics

**Electromagnetic Optics**: Solving Maxwell's equations for Anisotropic materials

**Wave Optics**: The interaction between plane waves or spherical waves. The structures are large compared to the wavelength. Materials are isotropic.

**Interference**

**Ray Optics**: Treat light as rays or lines that travel through a system. Light does not interact with other light.

Most design of optical systems is done with ray optics, Analysis of optical systems is a combination of ray and wave optics.

Ray optics is also called geometrical optics

Wave optics is also called physical optics
Ray Propagation
This is the high frequency limit. It ignores diffraction.
Each ray corresponds to a localized plane wave

Uniform beam

<table>
<thead>
<tr>
<th>Extent</th>
<th>Rays</th>
</tr>
</thead>
<tbody>
<tr>
<td>beam</td>
<td>p</td>
</tr>
</tbody>
</table>

planes of constant phase

Spherical Wave

Rays perpendicular to wave fronts

Each ray corresponds to a bucket of power. The spread of the rays corresponds to a decrease in power density.

Ray tracing is primarily done in incoherent systems, where if two rays hit it just has twice the power.

When a ray hits an interface Snell's Law is applied and then the ray travels in a straight line between interfaces. Since the light is incoherent the phase of each ray does not matter.

Good lens

All rays pass through the same point. Is the focal spot infinitesimally small?

Also useful in arbitrary optical systems.

Not valid if the dimensions are on the order of the wavelength.
Are the rays essentially narrow beams of light (like laser beams)? No. Very narrow beam diverge rapidly because of diffraction!

Rays are simply used to characterize the propagation of a wavefront. Even though rays are not actually narrow beams of light, they can still be used to analyze irradiance by looking at ray density.

Example

In 2-dimensions

\[
\begin{align*}
\text{source} & \quad \text{detector of width } D \\
P = \text{power} & \quad D = 1 \text{mm} \\
R = 1 \text{m} &
\end{align*}
\]

If you trace \( N \) equally spaced rays

Each ray has power \( P_r = \frac{P}{N} \)

\[
\Delta \theta = \frac{2 \pi}{N}
\]

Angle to the detector is \( \theta = \pm \tan^{-1} \left( \frac{0.5 \times 10^{-3}}{1} \right) = 0.5 \times 10^{-3} \)

Number of received rays

\[
\Delta \theta < \theta \\
\frac{2 \pi}{N} n < 0.5 \times 10^{-3}
\]

\[
n = \left( 0.5 \times 10^{-3} \right) \left( \frac{N}{2 \pi} \right) \quad \text{single side}
\]

\[
n = \left( 0.5 \times 10^{-3} \right) \left( \frac{N}{2 \pi} \right) \quad \text{double side}
\]

\[
\text{Prec} = (n)(P_r) = 10^{-3} \left( \frac{N}{10} \right) \left( \frac{1}{N} \right) = 10^{-3}
\]

If \( N = 10^5 \)

\[
n = \left( 10^{-3} \right) \left( \frac{10^5}{2 \pi} \right) = 15.92 \quad \text{so } 15
\]

\[
\text{Prec} = \frac{15}{10^{-3}} = 150 \text{ mW} \quad \text{with } N: \infty \quad \text{Prec} = 157.2 \text{ mW}
\]
Postulates of ray optics
1. Light travels in rays
2. Optical medium is characterized by the index of refraction
3. The travel time \( t = \frac{\text{OPL}}{c} \) depends on the optical path length
   \( \text{OPL} = n \text{d} \) or \( \text{OPL} = \int_A^B n(x) \text{d}l \)
4. Light rays travel along the path of least time
   Fermat's Principle

Example: Refraction at a planar interface

\[
\text{OPL} = \int_{P_1}^{P_2} n(x) \text{d}l = \int \left[ n_1, n_1 + n_2, n_2 \right] \text{d}l
\]

\[
\text{OPL} = n_1 \sqrt{d_1^2 + x^2} + n_2 \sqrt{d_2^2 + (L-x)^2}
\]

\[
\frac{d\text{OPL}}{dx} = \frac{n_1}{2} (d_1^2 + x^2)^{-\frac{1}{2}} x (2x) + \frac{n_2}{2} (d_2^2 + (L-x)^2)^{-\frac{1}{2}} (L-x)(-1) = 0
\]

\[
n_1 \sqrt{\frac{x}{d_1^2 + x^2}} = n_2 \sqrt{\frac{L-x}{d_2^2 + (L-x)^2}}
\]

\[
n_1 \frac{x}{e_1} = n_2 \frac{L-x}{e_2}
\]

\[
\sin \theta_1 = \frac{x}{e_1}, \quad \sin \theta_2 = \frac{L-x}{e_2}
\]

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2
\]

Snell's Law

This ray optics process can be used to trace rays through material that has a varying refractive index profile.

We will only be discussing interfaces with bulk media.

We will use the following:
1. Rays travel straight in homogeneous materials
2. Use Snell's Law at interfaces
3. Angle of incidence equals angle of reflection
Things that we can analyze with ray tracing

Mirrors
  Planar
  Parabolic
  Elliptical
  Spherical

Prisms
  Reflecting
  Refracting

Lenses
  Spherical
  Aspheric

Light pipes

Ghost images
  Rays split at glass interfaces

Scatter

Illumination
  Extended sources
  Head lamps

Sequential ray tracing
  From surface 1 → 2 → 3 ...
  Trace hundreds of rays

Non-sequential ray tracing
  A ray hits whatever object it wants
  Trace thousands to millions of rays

We won't be doing this.
Planar Mirror

Reflected rays appear to originate from a different location called the image.

Parabolic Mirror

Parallel rays all pass through a focal point.

Elliptical Mirror

2 foci; rays from 1 focus image to the other focus.

Figure 1.2-1 Reflection from a planar mirror.

Figure 1.2-2 Focusing of light by a paraboloidal mirror.

Figure 1.2-3 Reflection from an elliptical mirror.
Spherical mirror

Substantially easier to fabricate than other curved mirror shape.

Center of sphere focuses back to the center. This is not very useful.

This is much more common than parabolic reflectors.

If the rays make small angles the spherical reflector acts like a parabolic reflector.

This is because \( \sin \theta \approx \theta \) if \( \theta \) is small.

In this case \( f = \frac{R}{2} \)

This approximation is called the PARAXIAL APPROXIMATION.

This will also be used to analyze glass interfaces.

Initial optical design is done using the these paraxial rays.

A ray tracing program does not make this approximation.

\[ \text{Figure 1.2-4 Reflection of parallel rays from a concave sphere} \]

\[ \text{Figure 1.2-5 A spherical mirror approximates a paraboloidal mirror} \]
Let's start by looking at a single spherical surface.

Find the point \( P \) at which the ray crosses the axis. Fermat principle says the ray follows the minimum optical path length, using triangle \( SAC \):

\[
q^2 = b^2 + c^2 - 2bc \cos A
\]

\[
q_1 = \left[ R^2 + (R+S_1)^2 - 2(R)(R+S_1) \cos \phi \right]^{1/2}
\]

\[
q_2 = \left[ R^2 + (S_2 - R)^2 - 2(R)(S_2 - R) \cos \phi \right]^{1/2}
\]

\[
OPL = n_1 q_1 + n_2 q_2 = 1
\]

\[
OPL = n_1 \left[ R^2 + (R+S_1)^2 - 2R(R+S_1) \cos \phi \right] + n_2 \left[ R^2 + (S_2 - R)^2 + 2R(S_2 - R) \cos \phi \right]
\]

\[
\frac{dOPL}{dR} = 0 = \ldots
\]

\[
\frac{n_1 (S_1 + R)}{\left[ R^2 + (S_1 + R)^2 - 2R(S_1 + R) \cos \phi \right]^{1/2}} = \frac{n_2 (S_2 - R)}{\left[ R^2 + (S_2 - R)^2 + 2R(S_2 - R) \cos \phi \right]^{1/2}}
\]

Solve this equation for \( S_2 \) in terms of \( n_1, n_2, S_1, R \).

This is not trivial to solve, so we make the very harse paraxial approximation

\[
\cos \phi \approx 1, \quad \frac{R}{S_1} \approx 1, \quad \frac{R}{S_2} \approx 1
\]

(Ray tracing program do not make this approximation.)

\[
\frac{n_1 (S_1 + R)}{R - (S_1 + R)} = \frac{n_2 (S_2 - R)}{R + (S_2 - R)}
\]

Simplify to get

\[
\frac{n_2 - n_1}{R} = \frac{n_1}{S_1} + \frac{n_2}{S_2}
\]
Method 2

\[ (180 - \phi) + \theta_2 + \theta_3 = 180 \]
\[ \theta_3 = \phi - \theta_2 \]

\[ \theta_1 + \phi + \theta_5 = 180 \]
\[ \theta_5 = 180 - \theta_1 - \phi \]

\[ \theta_4 + \theta_5 = 180 \]
\[ \theta_4 = 180 - \theta_1 - \phi \]

\[ n_1 \sin \theta_4 = n_2 \sin \theta_3 \]

\[ n_1 \sin (\theta_1 + \phi) = n_2 \sin (\phi - \theta_2) \]

\[ n_1 \theta_1 + n_1 \phi = n_2 \phi - n_2 \theta_2 \]

Paraxial approximation
\[ \sin \theta \approx \theta \]

\[ \Delta \text{ is small} \]
\[ \theta_1 = \frac{h}{s_1}, \quad \phi = \frac{h}{r}, \quad \theta_2 = \frac{h}{s_2} \]

\[ n_1 \left( \frac{h}{s_1} \right) + n_1 \left( \frac{h}{r} \right) = n_2 \left( \frac{h}{s_2} \right) - n_2 \left( \frac{h}{s_2} \right) \]

\[ \frac{n_1}{s_1} + \frac{n_1 - n_2}{r} = - \frac{n_2}{s_2} \]

\[ \frac{n_2 - n_1}{r} = \frac{n_1}{s_1} + \frac{n_2}{s_2} \]
\[ \frac{n_1}{S_1} + \frac{n_2}{S_2} = \frac{n_2 - n_1}{R} \]

Set \( S_1 = \infty \)  
This would be a collimated wave  
All rays are parallel

\[ \frac{n_2}{S_2} = \frac{n_2 - n_1}{R} - \frac{n_1}{\infty} \]

\[ S_2 = \frac{n_2}{n_2 - n_1} R \]

Set \( S_2 = \infty \)  
This is collimated to the right

\[ \frac{n_1}{S_1} = \frac{n_2 - n_1}{R} \]

\[ S_1 = \frac{n_1}{n_2 - n_1} R \]

So all rays coming from a particular point become collimated

Now let's add another surface.  

We are also going to assume that the lens thickness is small.  
"Thin Lens"

The equation relating object to image is

\[ \frac{1}{S_o} + \frac{1}{S_i} = \frac{n_2-n_1}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f} \]

A positive value for \( S_o \) means that it lies to the left.

A positive value for \( S_i \) means that it lies to the right.

A positive value for \( R \) means that it curves to the left.

(For the image, \( R_1 \) is positive and \( R_2 \) is negative.)
\[
\frac{1}{s_2} = \frac{1}{f} - \frac{1}{s_1}
\]

\[s_1 = f \quad \frac{1}{s_2} = \frac{1}{f} - \frac{2}{f} = 0 \quad s_2 = \infty \] (Diverging)

\[s_1 = f \quad \frac{1}{s_2} = \frac{1}{f} - \frac{1}{f} = 0 \quad s_2 = \infty \]

\[s_1 > f \quad s_1 = 2f \quad \frac{1}{s_2} = \frac{1}{f} - \frac{1}{2f} \quad s_2 = 2f \] (Converging)

\[s_1 = \infty \quad \frac{1}{s_2} = \frac{1}{f} - \frac{1}{\infty} \quad s_2 = f \]

\[s_1 < 0 \quad s_1 = -f \quad \frac{1}{s_2} = \frac{1}{f} + \frac{1}{f} \quad s_2 = \frac{f}{2} \] (Converging)
\[ t < 0 \]

\[ S_1 = \frac{-1}{f} \]
\[ \frac{1}{S_2} = -\frac{1}{f} - \frac{1}{f} = -\frac{3}{f} \]
\[ S_2 = -\frac{3}{f} \]

\[ S_1 = \frac{1}{f} \]
\[ \frac{1}{S_2} = -\frac{1}{f} - \frac{2}{f} = -\frac{2}{f} \]
\[ S_2 = -\frac{2}{f} \]

\[ S_1 = \infty \]
\[ \frac{1}{S_2} = -\frac{1}{f} \]
\[ S_2 = -f \]

\[ S_1 = -\frac{1}{f} \]
\[ \frac{1}{S_2} = -\frac{1}{f} + \frac{1}{f} = 0 \]
\[ S_2 = \infty \]

\[ S_1 = -\frac{1}{f} \]
\[ \frac{1}{S_2} = -\frac{1}{f} + \frac{1}{f} = \frac{1}{f} \]
\[ S_2 = \frac{f}{2} \]

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\[ S_1 = \frac{1}{f} \]
\[ \frac{1}{S_2} = -\frac{1}{f} + \frac{1}{f} = \frac{1}{f} \]
\[ S_2 = f \]
Now let's look producing an image from an object using a single thin lens.

We will analyze the lens and image using the following properties:

1. A ray passing through the focal point is bent parallel to the axis of symmetry.

2. An incident ray parallel to the lens axis is bent to pass through the back focal point.

3. A ray passing through the center of the lens is unchanged in direction.

4. A point in the object space is imaged to a point in the image space.

5. Parallel rays image to a point. (A lens converts from incident angle to a point.)
Let's look at a finite object. An object is essential a collection of point sources in an object plane.

The image of the object is the plane where the collection of points occur.

The size of the image is determined by find the image of the top and bottom of the object.

First let's look at the location of the image.

\[
\frac{h_0}{f} = \frac{1}{d_0 - f} \]

Similar triangles \( \frac{h_0}{h_1} = \frac{S_0}{S_1} \)

combine this with above

\[
\frac{1}{f} = \frac{1}{S_0} + \frac{1}{S_1}.
\]
The transverse magnification is defined as

\[ M = \frac{h_1}{h_0} \]

\[ M = \frac{S_1}{S_0} \]

A positive \( M \) is an erect image.
A negative \( M \) is an inverted image.

Define \( x_0 = S_0 - f \)

\[ M = \frac{x_0}{f} \]
Let's look at collimated beams.

1. Draw line through lens center.
2. Draw line through front focal point.

All other lines go through the focus.

The focal points are at the same focal plane.

This is essentially an imager. Points in far-field space are focused to points on a focal plane imager.

\[ \tan \theta = \frac{D}{F} \quad \frac{D}{F} = \theta \]
Optical Terms

Radius of curvature \( R \)

Curvature \( C = \frac{1}{R} \)

dioptr : unit of curvature
\[ R = 10 \text{ cm} \]
\[ C = \text{cm}^{-1} = 10 \text{ D} \]

Focal length: the distance between the lens and image for a collimated beam (5000)

Lens power: \( P = f \) in dioptr

Accommodation: change in lens power of the human eye

Aberrations: difference between real ray position and the paraxial approximation

Magnification: ratio of image height to object height

Chief ray: ray that goes through the center of a lens

Thin lens equation: Equation that relates the image and object locations for an ideal thin lens

Aperture stop: the limiting aperture of the system

Pupil: the input or output aperture

Field of View (FOV): the angular extent of a focusing system

Field stop: the aperture that limits the FOV