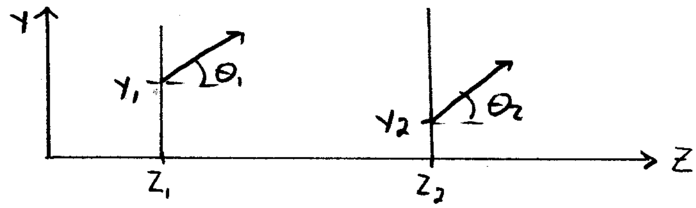
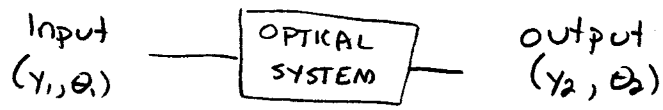
We want a simple way to analyze lens combinations using the paraxial approximation. This is done using ABCD matrices.

In a homogeneous material rays travel in straight lines. A ray is characterized by its vertical position and direction.





An optical system is a set of optical components that change the position and direction of rays.

In the paraxial approximation ($\sin\theta \approx \theta$). This makes the relationship between the input and output linear, resulting in the relation

$$y_2 = A y_1 + B \theta_1$$

$$\theta_2 = C y_1 + D \theta_1$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{M} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

The matrix M characterizes the optical system.

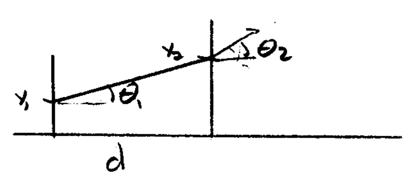
Let's calculate the matrices for some simple optical components.

<u>Free-space Propagation</u>

rays travel in straight lines

$$\theta_1 = \theta_2$$

$$y_2 = y_1 + \theta d$$



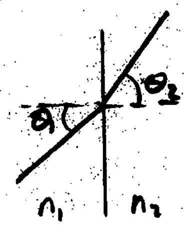
$$M = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Refraction at a Planar Boundary

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_1 \theta_1 \approx n_2 \theta_2$

$y_2 = y_1$
$\theta_2 = \frac{n_1}{n_2}\theta_1$ $\qquad M = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$

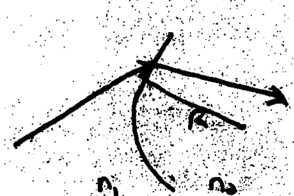


Refraction at a Spherical Boundary

$y_2 = y_1$

$\theta_2 \approx \frac{n_1}{n_2}\theta_1 - \frac{n_2 - n_1}{n_2 R} y_1$

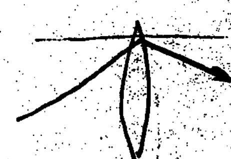
$$M = \begin{bmatrix} 1 & 0 \\ -\frac{n_2 - n_1}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$



Transmission through a Thin Lens

$y_2 = y_1$

$\theta_2 = \theta_1 - \frac{y}{f}$ $\qquad M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$



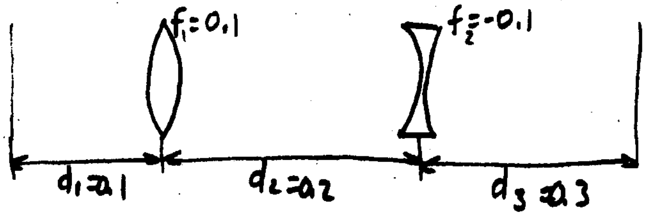
Reflection from a Planar Mirror

$y_2 = y_1$
$\theta_2 = \theta_1$ $\qquad M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Reflection from a Spherical Mirror

$$M = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

Multiple elements are analyzed by multipling all of the matrices for the individual elements.



$$M=\begin{bmatrix}1 & d_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -\frac{1}{f_1} & 1\end{bmatrix}\begin{bmatrix}1 & d_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -\frac{1}{f_2} & 1\end{bmatrix}\begin{bmatrix}1 & d_3\\ 0 & 1\end{bmatrix}$$

if the initial ray is $y_1=0$
$\theta_1 = 1^\circ = 0.0174$

$$\begin{bmatrix}y_2\\ \theta_2\end{bmatrix}=\begin{bmatrix}1 & .1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -10 & 1\end{bmatrix}\begin{bmatrix}1 & .2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 10 & 1\end{bmatrix}\begin{bmatrix}1 & .3\\ 0 & 1\end{bmatrix}\begin{bmatrix}0\\ .0174\end{bmatrix}=\begin{bmatrix}0.007\\ -.1218\end{bmatrix}$$

Can use this technique to perform a simple ray trace through an arbitrary system.

Let's look at Gaussian beam propagation through thin lenses.

The complex amplitude of a Gaussian beam is

$$U(r) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

When a wave propagates through a thin lens the field is modified by the complex transmittance of a lens

$$T(x,y) = \exp(-jnk_0 d) \exp\left[jk_0 \frac{\rho^2}{2f}\right]$$ (Eq 2.4-6 in book)

The lens just affects the phase portion of the Gaussian beam

$$kz + k\frac{\rho^2}{2R} - \zeta \Rightarrow kz + k\left(\frac{\rho^2}{2R} - \frac{\rho^2}{2f}\right) - \zeta + nkd$$

$$\phi = kz + k\frac{\rho^2}{2R'} - \zeta + nkd$$

where $\frac{1}{R'} = \frac{1}{R} - \frac{1}{f}$

The thin lens does not affect the Gaussian beam intensity profile but adjust the radius of curvature of the Gaussian phase. This results in a shift in the focus of the Gaussian beam.

A lens can shape the Gaussian beam without disrupting its Gaussian beam propagation characteristics.

We want to use lens to shape the Gaussian beam.

First let's review Gaussian beam propagation.

(1) The beam focus is the same thing as beam collimation.
(2) The depth of focus (length of collimation) depends on the beam waist

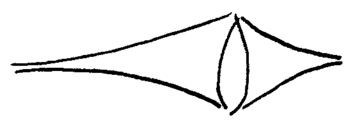
$$\text{small } W_o \rightarrow \begin{array}{l}\text{Small } 2z_o \\ \text{Large } \theta_o\end{array}$$

What do we typically want to do with a Gaussian beam?

(1) Produce the smallest spot as possible
(2) Produce a very collimated beam

Here are the primary beam shaping configurations:

(1) Diverging beam focused to a small spot

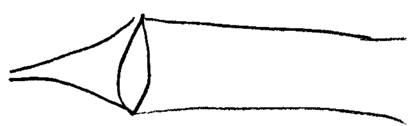


$z - f >> z_o$

(2) Collimated beam focused to a small spot

$z = 0$

(3) Diverging beam collimated



(4) Collimated beam made larger

Analyze just using ray tracing

(1) $Z - f \gg Z_0$ $\qquad r \approx 0$

$$M = M_r = \left|\frac{f}{Z-f}\right|$$

$$W_0' = \left|\frac{f}{Z-f}\right| W_0$$

In order to get the smallest spot, you want f as small as possible.

(2) $Z = 0$

$r = -\frac{Z_0}{f}$

$M_r = 1$

$M = \frac{1}{\sqrt{1+\left(\frac{Z_0}{f}\right)^2}}$

$$W_0' = \frac{W_0}{\sqrt{1+\left(\frac{Z_0}{f}\right)^2}}$$

To get W_0' small we want f small

Assume $f \ll Z_0$

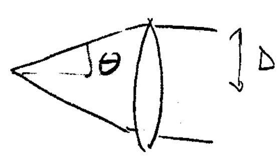
$$W_0' = \frac{W_0}{Z_0/f} = f\frac{W_0}{Z_0} = f\,(W_0)\frac{\lambda}{\pi W_0^2}$$

$$W_0' = \frac{f\lambda}{\pi W_0}$$

So you want $\frac{f}{W_0}$ small
So a small F#

(3) Look at this with ray tracing

$\theta = \frac{\lambda}{\pi W_0}$

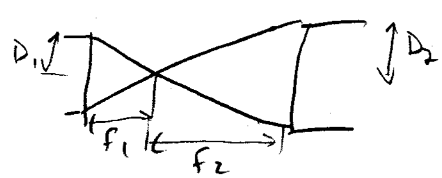


choose $\theta = \frac{D}{f}$

$f = \frac{D}{\theta} = D\frac{\pi W_0}{\lambda}$

(4) Look at this with ray tracing

$\frac{D_2}{D_1} = \frac{f_2}{f_1}$



We can also use ABCD matrices to analyze Gaussian beam propagation. This uses the q-parameter

$$\frac{1}{q} = \frac{1}{R} - j\frac{\lambda}{\pi W^2}$$

$$q_1 \rightarrow \boxed{\begin{array}{c}\text{OPTICAL}\\ \text{SYSTEM}\end{array}} \rightarrow q_2$$

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$

We will not be deriving this relationship in this class.

Using this equation we can use the previously derived ABCD matrices. Using ABCD matrices does not require any approximation on z, z_0, f as long as the paraxial approximation ($\sin\theta \approx \theta$) is valid.