

EE466 Optical Engineering Homework #4

1. (Book problem 9.2-1) A step-index fiber has radius $a=5\ \mu\text{m}$, core refractive index $n_1=1.45$, and fractional refractive-index change $\Delta=0.002$. Determine the shortest wavelength λ_c , for which the fiber is a single-mode waveguide. If the wavelength is changed to $\lambda_c/2$. Identify the indexes (l,m) of all the guided modes.
2. (Book problem 9.3-3) At $\lambda_o=820\ \text{nm}$ the absorption loss of a fiber is $0.25\ \text{dB/km}$ and the scattering loss is $2.25\ \text{dB/km}$. If the fiber is used instead at $\lambda_o=600\ \text{nm}$, and the calorimetric measurements of the heat generated by light absorption give a loss of $2\ \text{dB/km}$, estimate the total attenuation at $\lambda_o=600\ \text{nm}$.
3. (Book problem 9.3-4) Determine the core radius of a multimode step-index fiber with a numerical aperture $\text{NA}=0.1$ if the number of modes $M=5000$ when the wavelength is $0.87\ \mu\text{m}$. If the core refractive index $n_1=1.445$, the group index $N_1=1.456$, and Δ is approximately independent of wavelength, determine the modal-dispersion response time σ_τ for a $2\ \text{km}$ long fiber.
4. (Book problem 9.-5) Consider a graded-index fiber with $a/\lambda_o=10$, $n_1=1.45$, $\Delta=0.01$, and power-law profile with index p . Determine the number of modes M , the modal-dispersion pulse-broadening rate σ_τ/L , for $p=1.9, 2, 2.1$, and ∞ .
5. Derive the numerical aperture of a fiber, which is defined as $\text{NA}=\sin(\theta)$, where θ is the maximum ray angle that can be confined to the fiber. This equation should be a function of the refractive index of the core n_1 and the cladding (n_2).
6. The power measured out of an arbitrary length of optical fiber is $P=100\ \mu\text{W}$. A $10\ \text{m}$ section of optical fiber is then cut off. The power is then measured to be $105\ \mu\text{W}$. What is the attenuation of the optical fiber in dB/km ?
7. Standard single mode optical fiber has an attenuation of $\alpha=0.2\ \text{dB/km}$. The laser has a power of $P=5\ \text{mW}$. The coupling between the laser and the fiber is 70% . The coupling between the fiber and the detector is 90% . The fiber comes in $10\ \text{km}$ spools. Each splice between fiber spools results in a $0.3\ \text{dB}$ loss. What is the maximum link distance for which the received power is greater than $-22\ \text{dBm}$.
8. An optical fiber has the following parameters: $n_{\text{cladding}}=1.447$ and $\Delta=0.003$. The fiber needs to be single mode for both $1310\ \text{nm}$ and $1550\ \text{nm}$ wavelength lasers. What is the core diameter if the cutoff wavelength is designed to be $\lambda_c=1260\ \text{nm}$?
9. An optical fiber designed to be single mode at the operating wavelength of $\lambda=1550\ \text{nm}$ has the following parameters: $n_{\text{cladding}}=1.447$, $\Delta=0.003$, core diameter $d_{\text{core}}=9\ \mu\text{m}$. How many modes does it support if a red laser diode is ($\lambda=670\ \text{nm}$)?

10. Calculate the dispersion limited length for a data rate of 100Mbps for the following cases:
- A graded index multi-mode optical fiber (use the spec sheet posted on the class web site) with a single longitudinal mode Fabry-Perot laser with a linewidth of $\Delta\lambda = 2\text{nm}$.
 - An SMF28 single mode fiber (use the spec sheet posted on the class web site) with a multimode Fabry-Perot laser with modes every 107GHz that cover a total spectral width of $\Delta f = 5\text{THz}$.
 - An SMF28 single mode fiber (use the spec sheet posted on the class web site) with a single longitudinal mode Fabry-Perot laser with a linewidth of $\Delta\lambda = 2\text{nm}$.
 - An SMF28 single mode fiber (use the spec sheet posted on the class web site) with a DFB laser that has a spectral width of $\Delta f = 10\text{MHz}$.
11. What is the maximum data rate for a system designed with the following specifications
- Graded index multi-mode optical fiber
 - FP laser with a wavelength of $\lambda = 870\text{nm}$, a linewidth of $\Delta\lambda = 2\text{nm}$, and a power of 10mW.
 - Receiver with a minimum detectable power of $P_{\min} = -20\text{ dBm}$ at a data rate of 2.4Gbps. Minimum detectable power scales linearly with bit rate.
 - Link length of 3km
12. What is the maximum number of repeaters necessary for a link between New York and London (5585km) if the total data throughput is 10Gbps? The link uses SMF28 fiber, DFB lasers with a center wavelengths of $\lambda = 1550\text{nm}$ and spectral widths of $\Delta f = 10\text{MHz}$, transmission power of 10mW, and receivers with minimum detectable powers of $P_{\min} = -20\text{dBm}$ at a data rate of 2.4Gbps. Minimum detectable power scales linearly with bit rate. Remember that you need an individual repeater for each required fiber. Be sure to include both attenuation and dispersion limited length.
- Each fiber uses a data rate of 100Mbps
 - Each fiber uses a data rate of 1Gbps
 - Each fiber uses a data rate of 10Gbps