1. A achromat doublet (see Figure 1) from Newport has the following specifications:
   R_1=60.741 mm
   R_2=-44.710 mm
   R_3=-133.104 mm
   t_1=5 mm
   t_2=2.17 mm
   n_1=1.515 (@λ=633nm)
   n_2=1.668 (@λ=633nm)
   Using the matrix optics approach, what is the focal length of the lens?

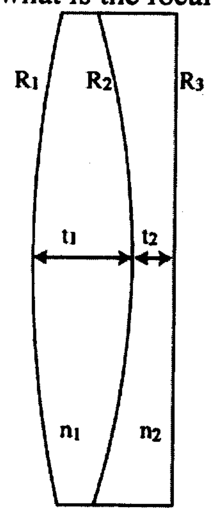


Figure 1: Lens doublet

Calculate the composite ABCD matrix

First surface $n_1 = 1$, $n_2 = 1.515$, $R = 60.741$ mm

$$M_1 = \begin{bmatrix} 1 & 0 \\ -\frac{(1.515-1)}{(1.515)(60.741)} & \frac{1.0}{1.515} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -5.596\times10^{-3} & .66 \end{bmatrix}$$

Propagation $$M_2 = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$$

2nd surface $n_1 = 1.515$, $n_2 = 1.668$, $R = -133.104$

$$M_3 = \begin{bmatrix} 1 & 0 \\ +\frac{(1.668-1.515)}{(1.668)(44.710)} & \frac{1.515}{1.668} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2.1\times10^{-3} & 0.908 \end{bmatrix}$$

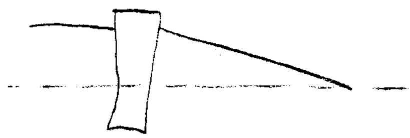
Propagation $$M_4 = \begin{bmatrix} 1 & 2.17 \\ 0 & 1 \end{bmatrix}$$

3rd surface $n_1 = 1.668$, $n_2 = 1.0$, $R = -133.104$

$$M_5 = \begin{bmatrix} 1 & 0 \\ +\frac{(1-1.668)}{133.104} & 1.668 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -5.02\times10^{-3} & 1.668 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 \\ -5.02\times10^{-3} & 1.668 \end{bmatrix} \begin{bmatrix} 1 & 2.7 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2.1\times10^{-3} & 0.908 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -5.6\times10^{-3} & 0.66 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.9653 & 4.616 \\ -0.01 & 0.9881 \end{bmatrix}$$



$$\begin{bmatrix} Y_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} Y_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} Y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0.9653 & 4.616 \\ -0.01 & 0.9881 \end{bmatrix} \begin{matrix} Y_0 \\ 0 \end{matrix}$$

$$= \begin{bmatrix} 0.9653\, Y_0 \\ -0.01\, Y_0 \end{bmatrix}$$

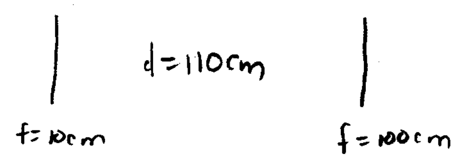
Find where Y goes to zero

$$Y = Y_2 + \theta_2 d$$
$$0 = 0.9653\, Y_0 - 0.01\, Y_0 f$$

$$f = \frac{0.9653}{+0.01} = 96.56\text{ mm}$$

$$\boxed{f \approx 100\text{mm}}$$

2. A 10x telescope consist of two positive lenses of focal lengths f_1=10cm and f_2=100cm separated by 110cm.
   a. What is the ABCD matrix for the optical system?
   b. A ray is incident onto the first lens of the telescope at a height of y_1=5mm and an angle of θ_1=5°. What is the height and angle of the ray after passing through the final lens?



$$M_1 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{10} & 1 \end{bmatrix} \quad M_2 = \begin{bmatrix} 1 & 110 \\ 0 & 1 \end{bmatrix} \quad M_3 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{100} & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 \\ -0.01 & 1 \end{bmatrix} \begin{bmatrix} 1 & 110 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -0.1 & 1 \end{bmatrix}$$

(a) $$M = \begin{bmatrix} -10 & 110 \\ 0 & -0.1 \end{bmatrix}$$

First check to see if it functions as a beam expander

$$\begin{matrix} y_1 \\ \theta_1 \end{matrix} = \begin{bmatrix} D \\ 0 \end{bmatrix} \qquad \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -10 & 110 \\ 0 & -0.1 \end{bmatrix} \begin{bmatrix} D \\ 0 \end{bmatrix}$$

$$\begin{matrix} y_2 \\ \theta_2 \end{matrix} = \begin{bmatrix} -10D \\ 0 \end{bmatrix}$$ 10x Magnification

(b) $$\begin{matrix} y_1 \\ \theta_1 \end{matrix} = \begin{bmatrix} 0.5\text{ cm} \\ 5\left(\frac{\pi}{180}\right) \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.087 \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -10 & 110 \\ 0 & -0.1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.087 \end{bmatrix} = \begin{bmatrix} -5 + 9.57 \\ -0.0087 \end{bmatrix} = \begin{bmatrix} 4.57 \\ -0.0087 \end{bmatrix}$$

$$\boxed{\begin{matrix} y_2 = 4.57\text{ cm} \\ \theta_2 = 0.5° \end{matrix}}$$

3. The telescope of the previous problem is misaligned by decreasing the separation of the lenses to 90cm. What are the height and angle of the following rays?
   y_1=5mm, θ_1=0
   y_1=0mm, θ_1=0
   y_1=-5mm, θ_1=0

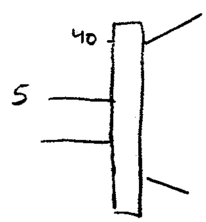
$$M = \begin{bmatrix} 1 & 0 \\ -0.01 & 1 \end{bmatrix} \begin{bmatrix} 1 & 90 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -0.1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 90 \\ -0.02 & 0.1 \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -8 & 90 \\ -0.02 & 0.1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} -40 \\ -0.1 \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -8 & 90 \\ -0.02 & 0.1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{matrix} y_2 \\ \theta_2 \end{matrix} = \begin{bmatrix} -8 & 90 \\ -0.02 & 0.1 \end{bmatrix} \begin{bmatrix} -5 \\ 0 \end{bmatrix} = \begin{bmatrix} 40 \\ 0.1 \end{bmatrix}$$



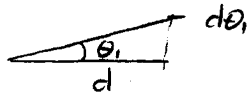
8× beam expansion

Not collimated 5.7° expanding beam

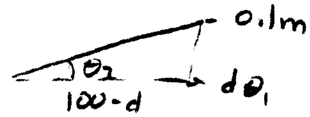
4. A source produces 5 narrow beams separated by 10° that are come from the same point ($y_1=0$ and $\theta_1=-20°, -10°, 0°, 10°, 20°$). These beams are designed to produce a set of dots on a screen located at a distance of L=100m away from the source. Design an optical system (a lens at a particular position) such that the beams produce dots on the screen that are separated by 10cm.

The system is (1) Free space propagation of distance d
(2) Thin lens of focal length f
(3) Free space propagation of distance. L-d

Do (1) and (3) by hand

(1) 

$Y_1 = d\theta_1$
$\theta_1 = \theta_1$

(2) 

$Y_2 = d\theta_1$
$\theta_1 = \theta_2$

$\theta_2 = \dfrac{0.1 - d\theta_1}{100-d}$

$$\begin{bmatrix} d\theta_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} d\theta_1 \\ \frac{0.1-d\theta_1}{100-d} \end{bmatrix}$$

$$\theta_1 = -\frac{d\theta_1}{f} + \frac{0.1 - d\theta_1}{100-d} \qquad \text{Where} \quad \theta_1 = (10)\frac{\pi}{180} = \frac{\pi}{18}$$

$$\frac{\pi}{18}\left[1 + \frac{d}{f} + \frac{d}{100-d}\right] = \frac{0.1}{100}$$

For a realistic system we will place the lens close to the point source so $d \ll 100$

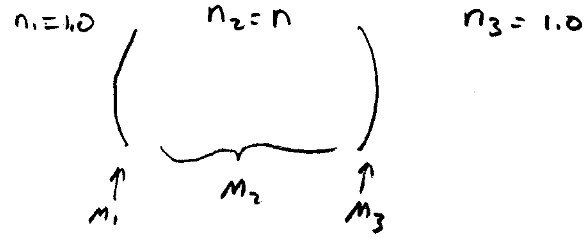
$$\frac{d}{100-d} \ll \frac{d}{f} \quad \text{and} \quad \frac{d}{100-d} \ll 1$$

$$\left[1 + \frac{d}{f}\right] = \frac{18}{1000\pi}$$

$$\frac{d}{f} = \frac{18}{1000\pi} - 1$$

$$\boxed{\frac{d}{f} = 0.994}$$

Derive the thin lens equation of 2 spherical surfaces separated by a distance d as d goes to zero.



$$M_1 = \begin{bmatrix} 1 & 0 \\ -\frac{(n-1)}{nR_1} & \frac{1}{n} \end{bmatrix} \qquad M_2 = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \qquad M_3 = \begin{bmatrix} 1 & 0 \\ -\frac{(1-n)}{R_2} & n \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 \\ \frac{(n-1)}{R_2} & n \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{(n-1)}{nR_1} & \frac{1}{n} \end{bmatrix}$$

let $d=0$

$$= \begin{bmatrix} 1 & 0 \\ \frac{(n-1)}{R_2} & n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{(n-1)}{nR_1} & \frac{1}{n} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ \frac{(n-1)}{R_2} & n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{(n-1)}{nR_1} & \frac{1}{n} \end{bmatrix}$$

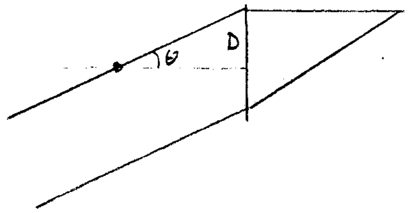
$$= \begin{bmatrix} 1 & 0 \\ \left(\frac{n-1}{R_2}\right) - \frac{(n-1)\,n}{n\,R_1} & 1 \end{bmatrix}$$

comparing to the thin lens ABCD matrix

$$-\frac{1}{f} = \left(\frac{n-1}{R_2}\right) - \left(\frac{n-1}{R_1}\right)$$

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

6. A square CCD array has a width of 1/3 inch. An imager is constructed using a single lens. The imager has a total field of view of 10° (-5°<θ<5°). The optical system has an f# equal to 5. What is the location and focal length of the lens?



$$\frac{D}{f} = \theta$$

$$f = \frac{D}{\theta} = \frac{1/6''}{5^\circ} = \frac{\frac{1}{6}\text{ in} \times \frac{25.4\text{ mm}}{1\text{ in}}}{5^\circ \times \frac{\pi}{180}\frac{\text{rad}}{\text{deg}}}$$

$$\boxed{f = 48.5\text{ mm}}$$

The CCD is located 48.5mm from the lens

The f# only affects the lens aperture size, which is

$$\frac{f}{D} = 5$$

$$D = \frac{f}{5} = \frac{48.5\text{ mm}}{5}$$

$$\boxed{D = 9.7\text{ mm}}$$

7. A Gaussian beam has a waist of W_o=0.1mm. This Gaussian beam is passed through the optical system shown in Figure 2. What is the Gaussian beam waist (W) at the ending point?

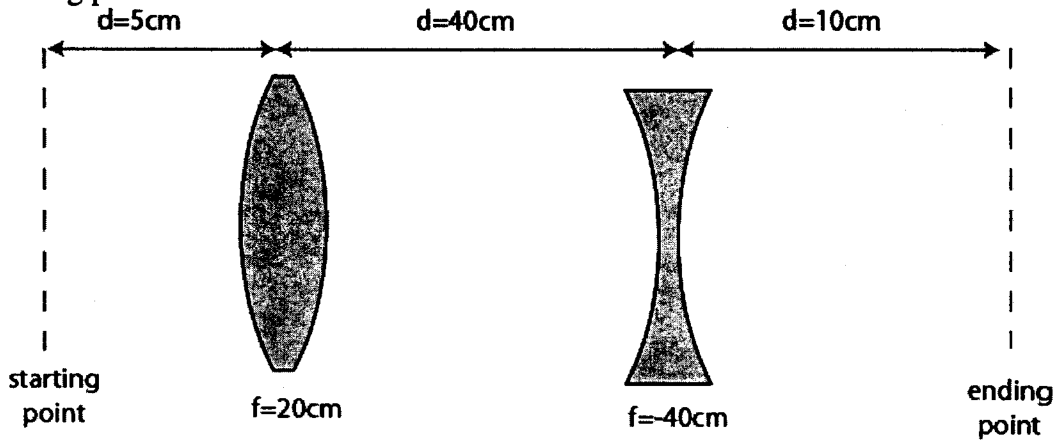


Figure 2

Calculate the ABCD matrix for the system

$M_1 = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ $M_2 = \begin{bmatrix} 1 & 0 \\ -0.05 & 1 \end{bmatrix}$ $M_3 = \begin{bmatrix} 1 & 40 \\ 0 & 1 \end{bmatrix}$ $M_4 = \begin{bmatrix} 1 & 0 \\ 0.025 & 1 \end{bmatrix}$

$M_5 = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$

$$M = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.025 & 1 \end{bmatrix} \begin{bmatrix} 1 & 40 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -0.05 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} -1.75 & 51.25 \\ -0.075 & 1.625 \end{bmatrix}$$

For the Gaussian beam at the starting point $W_0 = 0.1mm$

$R = \infty$

$\frac{1}{q} = \frac{1}{\infty} - j\frac{\lambda}{\pi(0.01)^2}$ keep units of cm

Assume $\lambda = 1 \mu m$

$\frac{1}{q_1} = -j\frac{1 \mu m \times \frac{1cm}{10^4 \mu m}}{\pi\ (0.01cm)^2} = -j\ 0.318$

$q_1 = j\pi$

$q_2 = \frac{(-1.75)j\pi + 51.25}{-0.075\,j\pi + 1.625} = 31.3695 + j\ 1.1652$

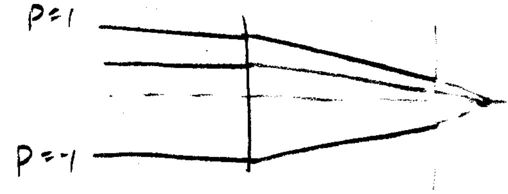
$\frac{1}{q_2} = 0.0318 - j0.0012 = \frac{1}{R} - j\frac{\lambda}{\pi W_0^2}$

$R = 1/0.0318 = \boxed{R = 31.45cm}$

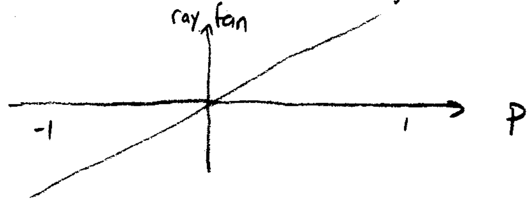
$\frac{\lambda}{\pi W_0^2} = 0.0012$ $W_0 = \sqrt{\frac{\lambda}{(\pi)(0.0012)}} = \boxed{0.164\ cm}$

~~$W_0 =$~~

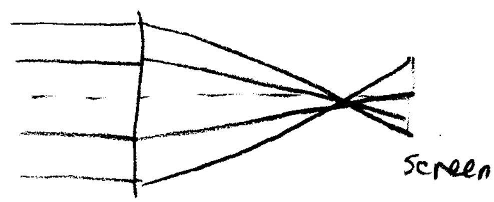
(8) (A) The distance between the lens and the screen is less than f



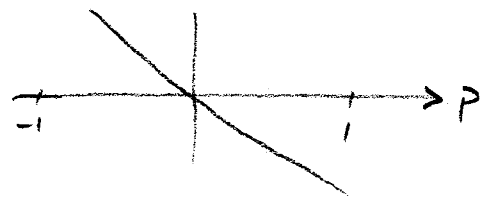
for p > 0 the height at the screen is greater than 0



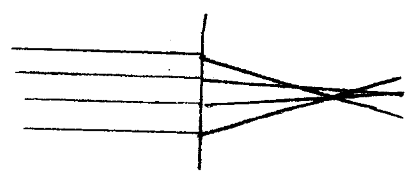
(B) The distance between the lens and the screen is greater than f



for p>0 the height at the screen is less than 0



(C) The distance is f but spherical aberation exists



← close to axis the height is small

P>0 height < 0